Class 3 – Self-organization in the Toroidal Pinch

⇒ to the RFP

Recall:

– early fusion rsch began with Z-pinch

– simplest closed magnetic confinement device

⇒ toroidal pinch (Zeta, late 50's

⇒ $a \sim 1\,M$, $I_\phi \sim 1$ MA)

⇒ idea: torus as secondary to transformer as primary.

⇒ toroidal current produces confinement and heating (ohmic)

but

⇒ stability ??

Stability Problem:

→  $\underline{F} = \underline{J} \times \underline{B} / c$

↓

force density

$$\rho \frac{d\underline{v}}{dt} = -\underline{\nabla} p + \frac{\underline{J} \times \underline{B}}{c}$$

$$2\pi r B_\theta = \frac{4\pi}{c} I$$

$$B_\theta = 2I/cr$$

$$f_r = -\frac{2I}{cr}\frac{J_\theta}{c} = -\frac{2I J_\theta}{c^2 r}$$

— sign for radially inward.

then $r \to a - \Delta$

$$f_r = \frac{-2I J_\theta}{c^2(a-\Delta)} \simeq -\frac{2I J_\theta}{c^2 a}\left[1+\frac{\Delta}{a}\right]$$

compression amplifier force

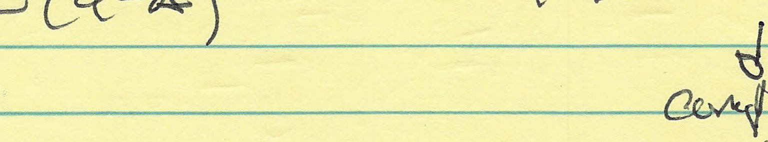
⇒ pinching force rises

- perturbation re-enforces itself
- instability!

→ "solution" — impose B_T

⇒ "stabilized pinch"

Physics:



Magnetic pressure vs magnetic tension.

i.e. δf_{pinch} vs $\delta f_{mag\ pressure}$ $\Rightarrow$ comparison of changes/increments

$$f_{pinch} = -\frac{2 I J_0}{c^2 a}\left[1 + \frac{\Delta}{a}\right]$$

$\uparrow$ eqbm

$$\delta f_{pinch} = -\frac{2 I J_0}{c^2 a}\left(\frac{\Delta}{a}\right) \hat{r}$$

$$\delta f_{pressure} = \delta\left(-\nabla_r P_{mag}\right) \hat{r}$$

$$P_{mag} = \frac{B_z^2}{8\pi}$$ $\rightarrow$ magnetic pressure (energy density)

$$-\nabla_r P_{mag} = \frac{B_z^2}{8\pi a} \hat{r}$$ $\rightarrow$ pushes outward

if squeezed: $a \rightarrow a - \Delta$

$$\delta f_{pressure} = \frac{B_z^2}{8\pi a}\left(\frac{\Delta}{a}\right)$$ $\rightarrow$ magnetic pressure rises.

8

$$\delta f = \delta f_{pres} + \delta f_{tens}$$

$$= -\frac{2 I J_\theta}{c^2 a}\frac{\Delta}{a} + \frac{B_z^2}{8\pi a}\frac{\Delta}{a}$$

competition

$$I_p \sim \frac{ca}{2\pi} B_\theta ,$$

$$J_\theta \sim \frac{c B_\theta}{2\pi^2 a}$$

$$\delta f = \frac{\Delta}{a}\left[\frac{B_z^2}{8\pi a} - \frac{2}{c^2 a}\,\frac{ca B_\theta}{2\pi}\,\frac{c B_\theta}{2\pi^2 a}\right]$$

$$= \frac{\Delta}{a}\left[\frac{B_z^2}{8\pi a} - \frac{B_\theta^2}{a\, 2\pi^2}\right]$$

net $\uparrow$ rise rate

$$\Rightarrow \boxed{B_z^2 > \frac{4}{\pi} B_\theta^2} \Rightarrow$$ critical B_z needed to stabilize pinch.

Aside :

- more precise heuristics,

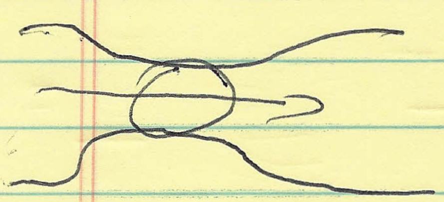
$$B_z^2 > 2 B_\theta^2$$

- origin : $\rho \frac{d\underline{v}}{dt} = -\underline{\nabla} p + \frac{\underline{J} \times \underline{B}}{c}$

$$= -\underline{\nabla}\left(p + \frac{\underline{B}^2}{8\pi}\right) + \frac{\underline{B}\cdot\underline{\nabla}\,\underline{B}}{4\pi}$$

$\downarrow$ pressure $\qquad \downarrow$ tension (only B_θ.

so



Pressure competition : δp_{B_z} vs δp_{B_θ}.

$$\delta(2\pi B_\theta r) = 0$$

(i.e. freeze in poloidal flux)

$$\delta B_\theta r + B_\theta \delta r = 0$$

$\left(A = 2\pi r L \quad \frac{}{L} r\right)$

$$-B_\theta \frac{\delta r}{r} = \delta B_\theta$$

$$\delta\left(\pi a^2 B_z\right) = 0$$

$$2\pi a\,\delta a\, B_z + \pi a^2 \delta B_z = 0$$

$$\delta B_z = -\frac{2\delta a}{a} B_z$$

So, need $\delta(B_z^2) > \delta(B_\theta^2)$

$$2 B_z\left(\frac{2\delta a}{a} B_z\right) > 2 B_\theta\left(B_\theta \frac{\delta a}{a}\right)$$

$$\boxed{B_z^2 > 2B_\theta^2}$$

→ Stabilized Pinch ⇒ Kinks!

→ Stabilized Pinch (misnomer/oxymoron)

- Kink turbulence state (not very stable)

$1\% < \delta B_\theta / B_\theta < 10\%$ → { Many MHD fluctuations

- $\tau_E \sim 100\ \mu$sec.

(bad confinement)
↓
chaos

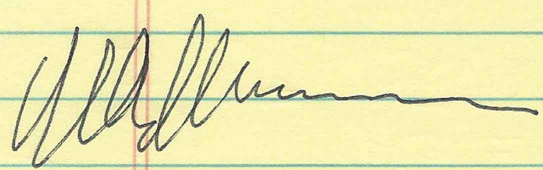
- $T_e \leq 100$ eV.

⇒ Looks bad for the Pinch . . . . .

but : $\Big\{$ The Quiescent Period

– early 60's

– for critical current :

→  $\delta B/B$ dropped.

→ $\bar{T}_e$ doubled ⇒ 150 – 200 eV.

→ τ_E increased → <u>1 msec</u>.

<u>and</u>

→ $B_T(r)$ "reversed" ⇒ hence <u>r</u>eversed <u>f</u>ield <u>p</u>inch

<u>i.e.</u>

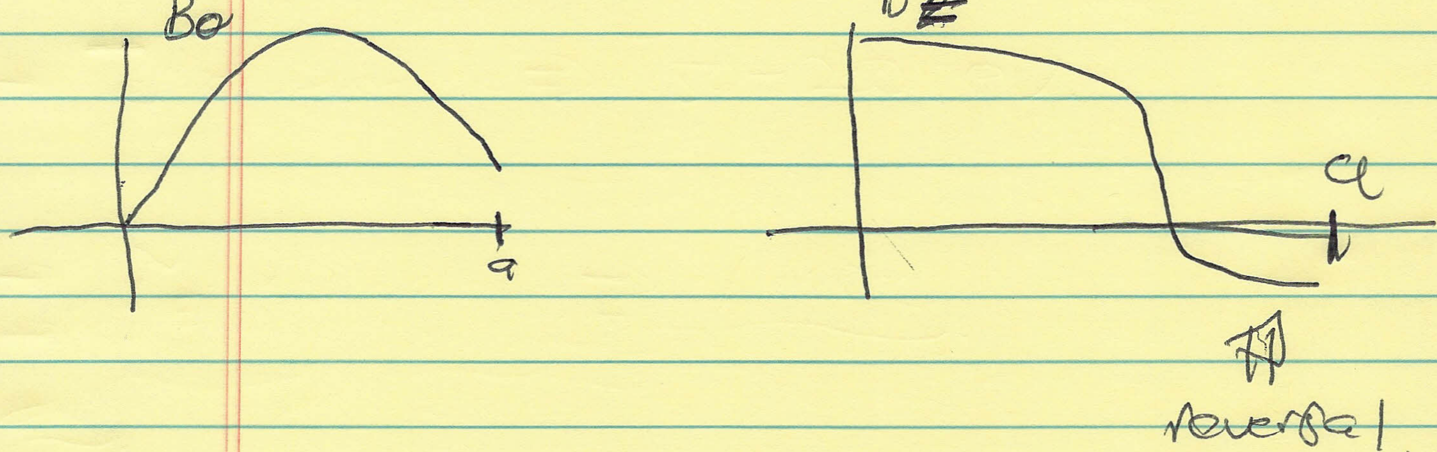
~ stabilized pinch J_T not aligned relative B_T i.e. B_T direction irrelevant

~ in RFP, $B_T J_T < 0$, $B_T E_T < 0$ } always at boundary.

field (B_T) reversed, relative to current, at boundary

characteristic of RFP (i.e. universal)

~ Plasma drives profiles:



~ what is remarkable here?

- plasma is producing B_z and reversal
- how??
  → transformer drives toroidal current
  → but plasma must, it seems,

N. B. Profiles are $\approx$ force free, i.e.

$$\underline{J} = \alpha \underline{B} \quad \text{so} \quad \underline{J} \times \underline{B} = 0$$

$$\underline{\nabla} \times \underline{B} = \frac{4\pi}{c} \underline{J} = \frac{4\pi}{c} \alpha \underline{B}$$

$$\underline{\nabla} \times \underline{\nabla} \times \underline{B} = \frac{4\pi}{c} \alpha \, \underline{\nabla} \times \underline{B}$$

$$-\nabla^2 \underline{B} = \left(\frac{4\pi}{c} \alpha\right)^2 \underline{B}$$

$$\boxed{\nabla^2 \underline{B} + \left(\frac{4\pi}{c} \alpha\right)^2 \underline{B} = 0}$$

Presumably such fields are <u>stable</u>.

drive poloidal current, to produce toroidal field?

Generic question:

→ How produce field parallel to current?

i.e

$$\xrightarrow{\quad\quad\underline{J}\quad\quad} \;\overset{?}{\Rightarrow}\; \xrightarrow{\quad\quad\underline{B}\quad\quad}$$

"Dynamo", here kink dynamo

→ dynamo: generic MHD phenomenon, system which amplifies magnetic field by turbulence

→ sun
→ earth
⋮

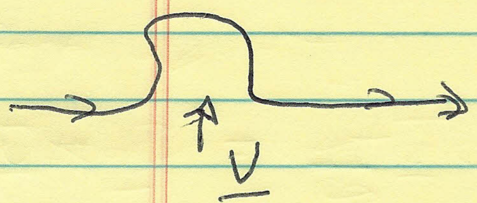
→ schematic cartoon (E.N. Parker, 1955)

$$\xrightarrow{\quad\quad\underline{J}\quad\quad}$$

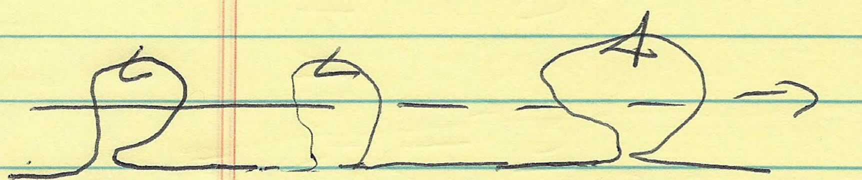
turbulent loops

then:

i.) deform loop



ii.) twist loop, correlate twists



$\underline{\omega} \rightarrow \underline{\omega} = \underline{\nabla} \times \underline{v}$

iii.) pinch off → (reconnection)

solenoid

Note:

a.) order in flow required

$\langle \underline{v} \cdot \underline{\omega} \rangle \neq 0$, over volume

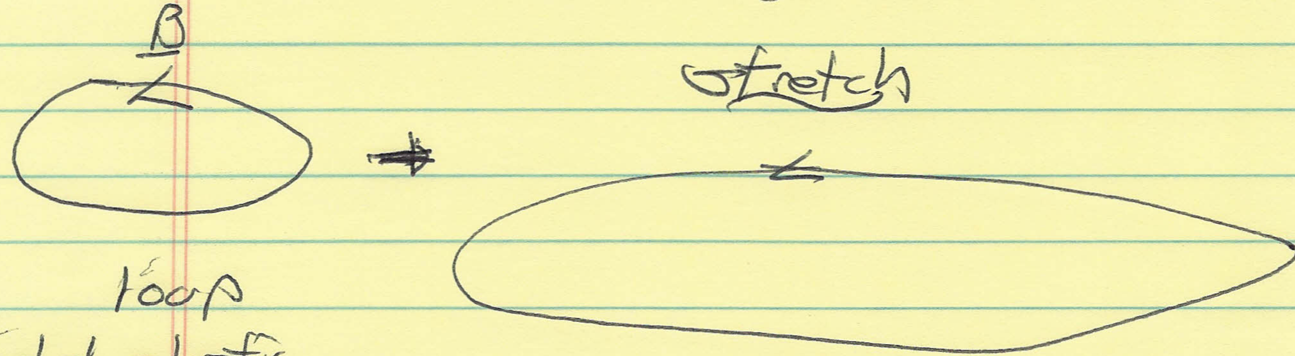
⇒ turbulence has net helicity

→ most generally,
- reflection symmetry breaking

b) "pinching off" – reconnection
- current layers/sheets

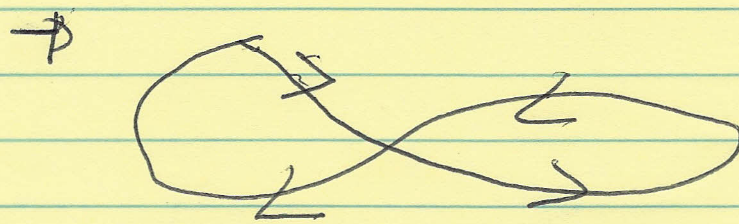
Process amplifies the field, i.e.

stretch

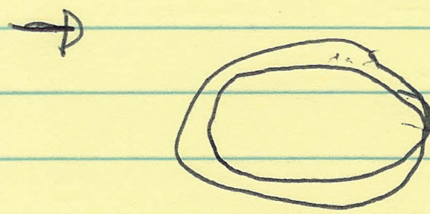


loop
(aka elastic band)

twist



fold



doubles tension

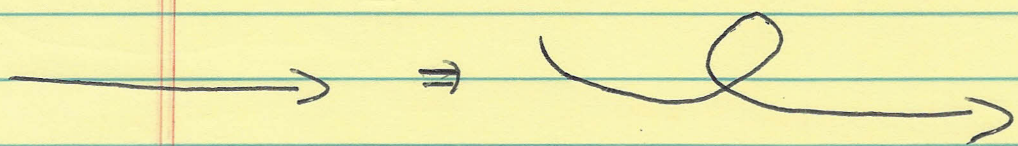
and reconnect → prevents coming undone

In RFP case:

→ not net amplification

→ rather process of converting toroidal current to poloidal current

→ turbulence is kink modes



– symmetry breaking from magnetic geometry

– locking-in ⇒ reversed surface ($m=0$).

What happens? "Magnetic Relaxation"
J.B. Taylor, 1974

→ instabilities ⇒ system seeks minimize energy

→ freezing-in law ⇒ some conservation / constraint on energy minimization.

Freezing-in ?

In ideal MHD, field "frozen-into" fluid, i.e. MHD as two inter-penetrating fluids. $(\underline{V}, \underline{B})$

i.e.
$$\left.\begin{array}{l} \underline{\nabla}\times\underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t} \\[2ex] \underline{E} + \frac{\underline{V}\times\underline{B}}{c} = \cancel{\eta \underline{J}} \end{array}\right\} \Rightarrow \partial_t \underline{B} = \underline{\nabla}\times(\underline{V}\times\underline{B}) + \eta\nabla^2\underline{B}$$

$$\Rightarrow \frac{d\,\underline{B}/\rho}{dt} = \frac{\underline{B}}{\rho}\cdot\underline{\nabla}\,\underline{V} + \cancel{}$$

$\underline{\nabla}\cdot\underline{V} = 0,$ $\qquad d\underline{B}/dt = \frac{\underline{B}}{\rho}\cdot\underline{\nabla}\,\underline{V}$

particles frozen in flow.

But observe:

$\bullet\ \underline{x}+\frac{\delta\underline{x}}{2}$

$\bullet\ \underline{x}-\frac{\delta\underline{x}}{2}$

$$\frac{d\underline{x}_1}{dt} = \underline{V}\left(\underline{x}+\frac{\delta\underline{x}}{2}, t\right)$$

$$\frac{d\underline{x}_2}{dt} = \underline{V}\left(\underline{x}-\frac{\delta\underline{x}}{2}, t\right)$$

$$\frac{d}{dt}(\underline{x}_1 - \underline{x}_2) = \underline{v}\left(\underline{x} + \frac{\delta \underline{x}}{2}, t\right) - \underline{v}\left(\underline{x} - \frac{\delta \underline{x}}{2}, t\right)$$

$$= \underline{v}(\underline{x}) + \frac{\delta \underline{x}}{2} \cdot \underline{\nabla}\,\underline{v} - v(\underline{x}) + \frac{\delta \underline{x}}{2} \cdot \nabla \underline{v}$$

$$\boxed{\frac{d}{dt}\delta \underline{x} = \delta \underline{x} \cdot \underline{\nabla}\,\underline{v}}$$

$\Rightarrow$ eqn. for length of line segment same as for $\underline{\underline{B}}$!

Now, related freezing in:

– conservation (constraint) $\int d^3x \; \underline{A} \cdot \underline{B}$

magnetic helicity

– self-linkage, self-inductance

– so $\delta\left(\int d^3x \left[\frac{B^2}{8\pi} + \lambda(\underline{A}\cdot\underline{B})\right]\right) = 0.$

$\Rightarrow \underline{J} = \lambda \underline{B}.$ force-free state emerges.

$$\frac{\underline{J}\cdot\underline{B}}{B^2} = \underline{\text{const.}}$$

RFP today:

→ basic quandry:
- dynamo 'good'

- but fluctuations ⇒ confinement degradation

RFP performance vastly inferior to tokamak, in confinement.

→ profile control (MST PPCD)
⇒ utility unclear

→ Another "good" surprise: QSH

~ Quasi Single Helicity State (Padova)

~ pinch kinks into single helix

~ confinement improved.